

C.U.SHAH UNIVERSITY

Summer Examination-2019

Subject Name: Mathematics - II

Subject Code: 4SC02MAT1

Branch: B.Sc. (All)

Semester: 2

Date: 29/04/2019

Time: 02:30 To 05:30

Marks: 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1 Attempt the following questions: (14)

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|-----------|---|---|
| a) | Find polar form of $(1 + i)$ | 1 |
| b) | If $y = \cos \theta + i \sin \theta$ then find value of $y + \frac{1}{y}$. | 1 |
| c) | If z is purely imaginary then $z \neq \bar{z}$. True/False. | 1 |
| d) | The number i^i is purely imaginary number. True/False. | 1 |
| e) | Define: Reciprocal cone. | 1 |
| f) | Write standard form of Ellipsoid. | 1 |
| g) | Solve: $(D - 1)^2 y = 0$ | 2 |
| h) | Find $\frac{1}{D-a} k$, where k is constant. | 2 |
| i) | Write tangency condition for cone. | 2 |
| j) | Prove that $\sin ix = i \sin hx$. | 2 |

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions (14)

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|-----------|--|---|
| a) | Consider the equation $(D^n + a_1 D^{n-1} + a_2 D^{n-2} + \dots + a_n)y = 0$ where a_i 's are constant. If m_1, m_2, \dots, m_n are real and different roots of A.E. Then prove that $y = c_1 e^{m_1 x} + c_2 e^{m_2 x} + \dots + c_n e^{m_n x}$ is complete solution of given equation. | 6 |
| b) | Find Particular Integral of $(D - 1)(D - 2)y = e^{-2x} + e^x + \sin 2x + \cos 3x$. | 4 |
| c) | Solve: $(D^4 - 1)y = e^x \cos x$. | 4 |

Q-3 Attempt all questions (14)

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|-----------|--|---|
| a) | Solve: $x^2 \frac{d^2 y}{dx^2} + 2x \frac{dy}{dx} - 20y = (x + 1)^2$. | 5 |
| b) | Solve: $(1 + x)^2 \frac{d^2 y}{dx^2} + (1 + x) \frac{dy}{dx} + y = 2 \sin \log(1 + x)$. | 5 |
| c) | Solve: $\frac{dx}{dt} + y = e^t, \frac{dy}{dt} + x = e^{-t}$. | 4 |

Q-4 Attempt all questions (14)

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|-----------|---|---|
| a) | Prove that $(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta \forall n \in \mathbb{Q}$. | 5 |
| b) | Solve: $(D^2 - 1)y = \cos hx \cos x$. | 5 |



- c) Prove that $\frac{1}{D-m} X = e^{mx} \int e^{-mx} X dx.$ 4
- Q-5 Attempt all questions (14)**
- a) Prove that $\left(\frac{1+\sin\theta+i\cos\theta}{1+\sin\theta-i\cos\theta}\right)^n = \cos\left(\frac{n\pi}{2} - n\theta\right) + i \sin\left(\frac{n\pi}{2} - n\theta\right).$ 5
- b) Solve: $x^4 - x^3 + x^2 - x + 1 = 0.$ 5
- c) Prove that $(a + ib)^{\frac{m}{n}} + (a - ib)^{\frac{m}{n}} = 2(a^2 + b^2)^{\frac{m}{2n}} \cos\left(\frac{m}{n} \tan^{-1} \frac{b}{a}\right).$ 4
- Q-6 Attempt all questions (14)**
- a) Expand $\sin^5 \theta$ in a series of sine multiple of $\theta.$ 5
- b) If $\cos^{-1}(u + iv) = x + iy$ then prove that 5
- (1) $u^2 \sec^2 x - v^2 \operatorname{cosec}^2 x = 1.$
- (2) $u^2 \sec^2 y + v^2 \operatorname{cosec}^2 y = 1.$
- c) Find real and imaginary part of $(i)^i.$ 4
- Q-7 Attempt all questions (14)**
- a) Prove that equation of cone which passes through (α, β, γ) and having guiding curve conic is 6
- $$a(\alpha z - x\gamma)^2 + b(\beta z - y\gamma)^2 + c(z - \gamma)^2 + 2h(\alpha z - x\gamma)(\beta z - y\gamma) + 2g(\alpha z - x\gamma)(z - \gamma) + 2f(\beta z - y\gamma)(z - \gamma) = 0.$$
- b) Find equation of cone whose vertex is $(-1, -2, -3)$ and base curve is $x^2 + z^2 = 1, y = 0.$ 5
- c) Identify the surface $x^2 + y^2 + z^2 + 4x - 6y = 3.$ 3
- Q-8 Attempt all questions (14)**
- a) Prove that equation of right circular cylinder having axis line $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$ and radius r is 6
- $$(x - \alpha)^2 + (y - \beta)^2 + (z - \gamma)^2 - \frac{[l(x - \alpha) + m(y - \beta) + n(z - \gamma)]^2}{l^2 + m^2 + n^2} = r^2.$$
- b) Find equation of cylinder whose generators are parallel to $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and having guiding curve $x^2 + y^2 = 16, z = 0.$ 5
- c) Find reciprocal cone of $ax^2 + by^2 + cz^2 = 0.$ 3

